



Geometry

SECOND EDITION

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The background of the top half of the cover is a complex geometric diagram. It features several overlapping circles and arcs, some solid and some dashed. Labels include 'R1', 'R2', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I', 'J', 'K', 'L', 'M', 'N', 'O', 'P', 'Q', 'R', 'S', 'T', 'U', 'V', 'W', 'X', 'Y', 'Z', '1', '2', '3', '4', '5', '6', '7', '8'. The diagram appears to be a technical drawing or a proof related to geometry, possibly involving the construction of a circle or a specific geometric property.

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In memory of Wilson Stothers

Contents

<i>Preface</i>	<i>page xi</i>
0 Introduction: Geometry and Geometries	1
1 Conics	5
1.1 Conic Sections and Conics	6
1.2 Properties of Conics	23
1.3 Recognizing Conics	36
1.4 Quadric Surfaces	42
1.5 Exercises	52
Summary of Chapter 1	55
2 Affine Geometry	61
2.1 Geometry and Transformations	62
2.2 Affine Transformations and Parallel Projections	70
2.3 Properties of Affine Transformations	84
2.4 Using the Fundamental Theorem of Affine Geometry	93
2.5 Affine Transformations and Conics	108
2.6 Exercises	117
Summary of Chapter 2	121
3 Projective Geometry: Lines	127
3.1 Perspective	128
3.2 The Projective Plane \mathbb{RP}^2	137
3.3 Projective Transformations	151
3.4 Using the Fundamental Theorem of Projective Geometry	172
3.5 Cross-Ratio	179
3.6 Exercises	192
Summary of Chapter 3	195
4 Projective Geometry: Conics	201
4.1 Projective Conics	202
4.2 Tangents	216
4.3 Theorems	229

4.4	Applying Linear Algebra to Projective Conics	248
4.5	Duality and Projective Conics	250
4.6	Exercises	252
	Summary of Chapter 4	255
5	Inversive Geometry	261
5.1	Inversion	262
5.2	Extending the Plane	276
5.3	Inversive Geometry	295
5.4	Fundamental Theorem of Inversive Geometry	310
5.5	Coaxal Families of Circles	317
5.6	Exercises	331
	Summary of Chapter 5	335
6	Hyperbolic Geometry: the Poincaré Model	343
6.1	Hyperbolic Geometry: the Disc Model	345
6.2	Hyperbolic Transformations	356
6.3	Distance in Hyperbolic Geometry	367
6.4	Geometrical Theorems	383
6.5	Area	401
6.6	Hyperbolic Geometry: the Half-Plane Model	412
6.7	Exercises	413
	Summary of Chapter 6	417
7	Elliptic Geometry: the Spherical Model	424
7.1	Spherical Space	425
7.2	Spherical Transformations	429
7.3	Spherical Trigonometry	438
7.4	Spherical Geometry and the Extended Complex Plane	450
7.5	Planar Maps	460
7.6	Exercises	464
	Summary of Chapter 7	465
8	The Kleinian View of Geometry	470
8.1	Affine Geometry	470
8.2	Projective Reflections	475
8.3	Hyperbolic Geometry and Projective Geometry	477
8.4	Elliptic Geometry: the Spherical Model	482
8.5	Euclidean Geometry	484
	Summary of Chapter 8	486
	<i>Special Symbols</i>	488
	<i>Further Reading</i>	490
	<i>Appendix 1: A Primer of Group Theory</i>	492