Mechanical science for technicians volume 1

Second edition

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Preface

In the second edition of this book, the text has been revised to meet the requirements of the new Business & Technician Education Council standard unit Mechanical science III (U82/040), the aim of which is to extend the student's understanding of mechanical engineering and to develop an analytical approach to the solution of problems involving deformation of materials, dynamics, and fluid flow. Material has been added on shear force and bending moment, and there are also new or revised sections on centroids, first and second moments of area, assumptions made in the theories of bending and torsion, static balancing, and flow through sharp-edged orifices.

SI units have been used throughout in the text, with the following preferred multiples and submultiples:

Prefix	*Symbol*	*Multiplication factor*
giga	G	10^9 = 1 000 000 000
mega	M	10^6 = 1 000 000
kilo	k	10^3 = 1000
milli	m	10^{-3} = 0.001
micro	μ	10^{-6} = 0.000 001

Ian McDonagh

1 Stress, strain, and elasticity

1.1 Types of force

There are three types of force which may be applied to a material:

i) tensile (or stretching) force, fig. 1.1(a);
ii) compressive (or squeezing) force, fig. 1.1(b);
iii) shear (or sliding) force, fig. 1.1(c).

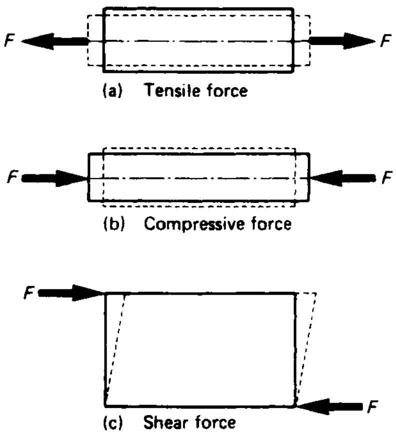

Fig. 1.1 Types of force

Tensile and compressive forces are *direct* forces. Direct forces are also known as *uniaxial* forces, since the opposing forces are in line, and these induce *direct* stresses in the material (see section 1.2). Shear forces are *indirect*, since the lines of action of the opposing forces must be separated as shown in fig. 1.1(c) for shear to occur. Shear forces induce *shear* stresses in the material (see section 1.8).

1.2 Direct stress

Direct stress is defined as the applied force F per unit cross-sectional area A resisting the force,

$$\text{i.e.}\quad \text{direct stress} = \frac{\text{applied force}}{\text{cross-sectional area resisting the force}}$$

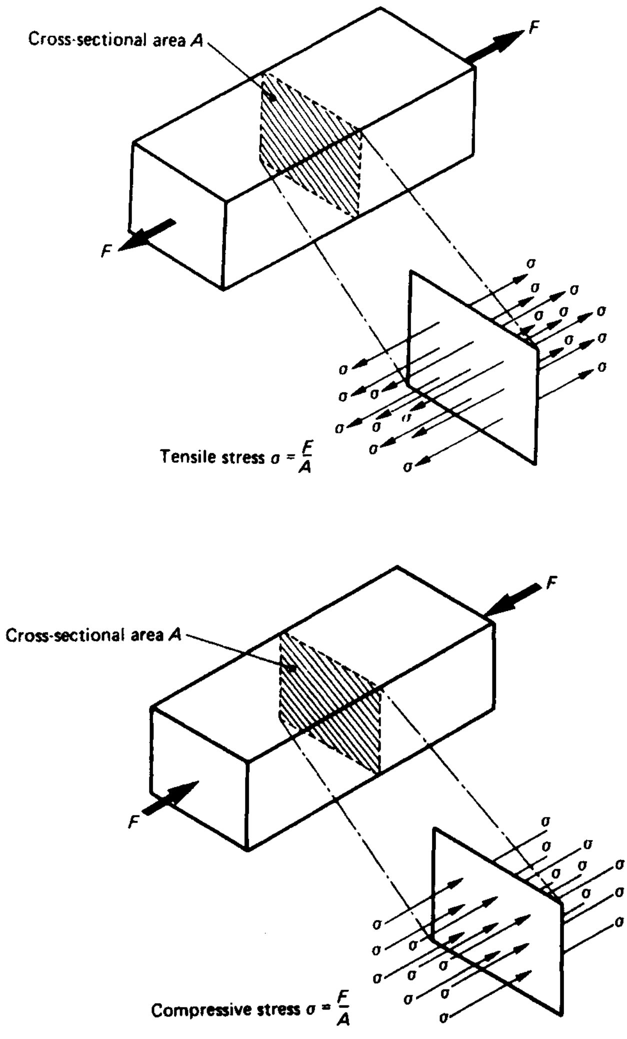

Fig. 1.2 Tensile and compressive stress

Direct stresses may be tensile or compressive, as illustrated in fig. 1.2. The symbol used for direct stress is σ (*sigma*); thus

$$\text{direct stress } \sigma = \frac{F}{A}$$

which should be remembered.

The basic unit for stress is the newton per square metre (N/m^2). Since this is very small, the unit meganewton per square metre (MN/m^2) is often used, and the units newton per square millimetre (N/mm^2) and *pascal* (Pa) may also be used. It is useful to remember that

$$1\ N/mm^2 = 1\ MN/m^2$$

and $$1\ Pa = 1\ N/m^2$$

1.3 Strain

Strain is defined as change in dimension (x) per unit original dimension (l),

i.e. $$\text{strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

Strain may be tensile, compressive, or shear. Tensile strain occurs when there is an *increase* in the original dimension, and compressive strain when there is a *decrease*. Shear strain is discussed in section 1.9.

The symbol used for tensile or compressive strain is ϵ (*epsilon*),

$$\therefore \quad \epsilon = \frac{x}{l}$$

which should be remembered.

Since strain is a ratio of *like* quantities, *it has no units.*

1.4 Modulus of elasticity

All solid materials will change shape slightly when stressed. If a material reverts back to its original shape and size when the stress is removed, it is an *elastic* material. Most solid materials are elastic up to a certain stress limit known as the *elastic limit* – a common exception to this is lead at room temperature. The stress–strain graph for an elastic material in fig. 1.3 shows

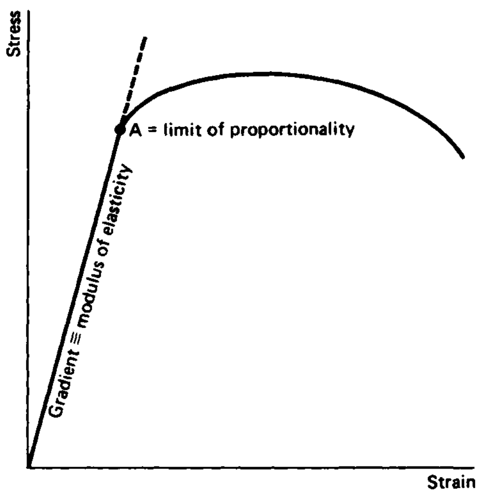

Fig. 1.3 Stress–strain graph for a material

that, up to a point which may be at or just below the elastic limit, the graph is a straight line; i.e., up to the point A (the *limit of proportionality*) in fig. 1.3, the stress is directly proportional to the strain, or

$$\sigma \propto \epsilon$$

$$\therefore \quad \frac{\sigma}{\epsilon} = \text{constant}$$

This constant is known as the *modulus of elasticity* or *Young's modulus*, E, for the material,

i.e. $\text{modulus of elasticity} = \dfrac{\text{stress}}{\text{strain}}$

or $E = \dfrac{\sigma}{\epsilon}$

which should be remembered.

The basic unit for E is the same as for stress, i.e. the newton per square metre (N/m^2), though the multiple giganewton per square metre (GN/m^2) is often used.

Table 1.1 shows typical values of the modulus of elasticity for various materials.

Table 1.1 Typical values of the modulus of elasticity. (Note that the modulus of elasticity of carbon steel is little altered for variations in the carbon content.)

Material	Modulus of elasticity (N/m^2)
Carbon steel	210×10^9
Copper	120×10^9
Cast iron	100×10^9
Brass	90×10^9
Aluminium alloy	90×10^9

Example In a tensile test on a steel sample, an extensometer recorded an increase in length of 0.117 mm for an applied load of 60 kN. If the diameter and the original gauge length of the sample were 15.96 mm and 80 mm respectively, determine the modulus of elasticity for the steel.

$$E = \sigma/\epsilon$$

but $\sigma = F/A$ and $\epsilon = x/l$

$$\therefore \quad E = \frac{Fl}{Ax}$$

which it is useful to remember.

Here, $F = 60\,kN = 60 \times 10^3\,N$ $\quad l = 80\,mm \quad x = 0.117\,mm$

and $A = (\pi/4) \times (15.96 \times 10^{-3}\,m)^2 = 200 \times 10^{-6}\,m^2$

$$\therefore \quad E = \frac{60 \times 10^3\,\text{N} \times 80\,\text{mm}}{200 \times 10^{-6}\,\text{m}^2 \times 0.117\,\text{mm}}$$

$$= 205.1 \times 10^9\,\text{N/m}^2$$

i.e. the modulus of elasticity of the steel is 205.1 GN/m^2.

1.5 Stress and strain in composite bars

Consider a composite bar consisting of a steel tube which is completely filled with rubber, as shown in fig. 1.4(a). If the composite bar is subjected to a compressive force F applied through the flat plates on the ends of the tube, as shown in fig. 1.4(b), then the *whole* assembly will suffer a *decrease* in length x, the decrease being limited by the stiffness of the stronger material, i.e. the steel tube. Also, the compressive force F is *shared* between the tube and the rubber, with the tube, being the more rigid, carrying most of the force. Thus, to solve problems relating to composite bars of this type, it should be remembered that

a) decrease in length of tube = decrease in length of rubber

or compressive strain in tube = compressive strain in rubber

b) total force = force on tube + force on rubber

Let subscript a refer to the tube and subscript b to the rubber, then

$$\epsilon_a = \epsilon_b \quad \text{and} \quad F = F_a + F_b$$

But strain $\epsilon = \sigma/E$ and force $F = \sigma A$

$$\therefore \quad \frac{\sigma_a}{E_a} = \frac{\sigma_b}{E_b} \qquad \text{(i)}$$

$$\text{and} \quad F = \sigma_a A_a + \sigma_b A_b \qquad \text{(ii)}$$

which are useful to remember.

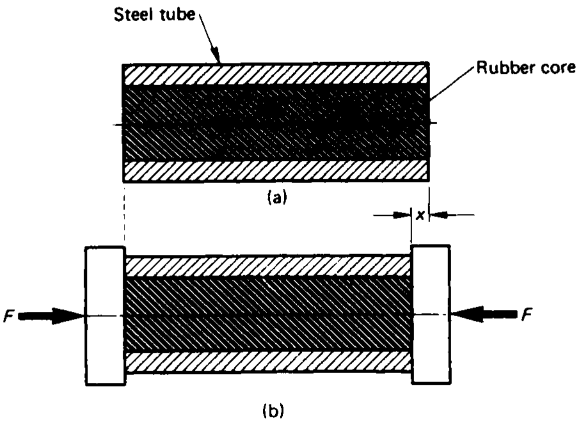

Fig. 1.4 Composite bar

Example 1 If the internal and external diameters of the steel tube shown in fig. 1.4 are 24 mm and 30 mm respectively, determine the compressive stresses in the rubber and in the steel when the applied force is 3 kN. For steel, $E = 200\,\text{GN/m}^2$; for rubber, $E = 2.5\,\text{GN/m}^2$.

Let subscript a refer to the steel tube and subscript b to the rubber.

$$\frac{\sigma_a}{E_a} = \frac{\sigma_b}{E_b}$$

$$\therefore \quad \sigma_a = \sigma_b\,(E_a/E_b)$$

where $E_a = 200\,\text{GN/m}^2$ and $E_b = 2.5\,\text{GN/m}^2$

$$\therefore \quad \sigma_a = \sigma_b \times \frac{200\,\text{GN/m}^2}{2.5\,\text{GN/m}^2} = 80\,\sigma_b$$

Also, $F = \sigma_a A_a + \sigma_b A_b = 80\,\sigma_b A_a + \sigma_b A_b$

where $F = 3\text{kN} = 3 \times 10^3\,\text{N}$

$A_a = (\pi/4) \times (30^2 - 24^2)\,\text{mm}^2 = 254.5\,\text{mm}^2$

and $A_b = (\pi/4) \times (24\,\text{mm})^2 = 452.4\,\text{mm}^2$

$$\therefore \quad 3 \times 10^3\,\text{N} = 80\,\sigma_b \times 254.5\,\text{mm}^2 + \sigma_b \times 452.4\,\text{mm}^2$$

$$= 2.081 \times 10^4\,\sigma_b\,\text{mm}^2$$

$$\therefore \quad \sigma_b = \frac{3 \times 10^3\,\text{N}}{2.081 \times 10^4\,\text{mm}^2}$$

$$= 0.144\,\text{N/mm}^2 \quad \text{or} \quad 0.144\,\text{MN/m}^2$$

$$\sigma_a = 80\,\sigma_b$$

$$\therefore \quad \sigma_a = 80 \times 0.144\,\text{MN/m}^2$$

$$= 11.52\,\text{MN/m}^2$$

i.e. the stresses in the rubber and the steel tube are $0.144\,\text{MN/m}^2$ and $11.52\,\text{MN/m}^2$ respectively.

Example 2 A cast-iron tube, 3 m long, is completely filled with concrete and used as a vertical strut. If the external diameter of the tube is 450 mm and the wall is 35 mm thick, determine the maximum compressive load the composite strut can support if the stress in the concrete is not to exceed $2\,\text{N/mm}^2$. By how much will the strut shorten under this load? For cast iron, $E = 100\,\text{GN/m}^2$; for concrete, $E = 10\,\text{GN/m}^2$.

Let subscript a refer to the cast iron and subscript b to the concrete; then,

strain in cast iron = strain in concrete

or $\sigma_a/E_a = \sigma_b/E_b$

$$\therefore \quad \sigma_a = \sigma_b\,(E_a/E_b)$$

where $\sigma_b = 2\,N/mm^2$ $\quad E_a = 100\,GN/m^2$ and $E_b = 10\,GN/m^2$

$$\therefore \quad \sigma_a = 2\,N/mm^2 \times \frac{100\,GN/m^2}{10\,GN/m^2}$$

$$= 20\,N/mm^2$$

Also, total force = force on cast iron + force on concrete

or $$F = \sigma_a A_a + \sigma_b A_b$$

where $A_a = (\pi/4) \times (450^2 - 380^2)\,mm^2 = 45.6 \times 10^3\,mm^2$

and $A_b = (\pi/4) \times (380\,mm)^2 = 113.4 \times 10^3\,mm^2$

$$\therefore \quad F = (20\,N/mm^2 \times 45.6 \times 10^3\,mm^2) + (2\,N/mm^2 \times 113.4 \times 10^3\,mm^2)$$

$$= 1.14 \times 10^6\,N \quad \text{or} \quad 1.14\,MN$$

i.e. the maximum force the composite strut can support is 1.14 MN.

Strain $\epsilon = x/l = \sigma/E$

$$\therefore \quad x = l(\sigma/E)$$

where $l = 3\,m$ and, for concrete, $\sigma = 2\,N/mm^2 = 2 \times 10^6\,N/m^2$

and $E = 10 \times 10^9\,N/m^2$

$$\therefore \quad x = 3\,m \times \frac{2 \times 10^6\,N/m^2}{10 \times 10^9\,N/m^2}$$

$$= 6 \times 10^{-4}\,m \quad \text{or} \quad 0.6\,mm$$

i.e. the composite strut will shorten by 0.6 mm.

Example 3 A concrete column of square cross-section, 250 mm x 250 mm, is required to support an axial load of 875 kN. Determine the minimum number of steel rods, each of diameter 6 mm, which would be required to reduce the stress in the concrete to $8\,N/mm^2$. For steel, $E = 200\,GN/m^2$; for concrete, $E = 12\,GN/m^2$.

Let subscript a refer to the steel, subscript b refer to the concrete, and n be the number of steel rods required.

Strain in steel = strain in concrete

or $$\sigma_a/E_a = \sigma_b/E_b$$

$$\therefore \quad \sigma_a = \sigma_b\,(E_a/E_b)$$

where $\sigma_b = 8\,N/mm^2$ $\quad E_a = 200\,GN/m^2$ and $E_b = 12\,GN/m^2$

$$\therefore \quad \sigma_a = 8\,N/mm^2 \times \frac{200\,GN/m^2}{12\,GN/m^2}$$

$$= 133.3\,N/mm^2$$

Total force = force on steel + force on concrete

or $$F = \sigma_a A_a + \sigma_b A_b$$

where $F = 875\,\text{kN} = 875 \times 10^3\,\text{N}$

$A_a = (\pi/4) \times (6\,\text{mm})^2 n = (28.27\,n)\,\text{mm}^2$

and $A_b = (250\,\text{mm})^2 - A_a = (62.5 \times 10^3 - 28.27\,n)\,\text{mm}^2$

$$\therefore \quad 875 \times 10^3\,\text{N} = 133.3\,\text{N/mm}^2 \times (28.27\,n)\,\text{mm}^2 + 8\,\text{N/mm}^2 \times (62.5 \times 10^3 - 28.27\,n)\,\text{mm}^2$$

$$\therefore \quad 875 \times 10^3 = 3768.4\,n + 500 \times 10^3 - 226.2\,n$$

from which, $n = 105.9$

i.e. the minimum number of steel rods required is 106.

1.6 Stresses due to temperature change

A solid material expands when its temperature is increased and contracts when its temperature is decreased. The change in dimension, x, which occurs with change in temperature is given by

$$x = l\,\alpha\,\Delta\theta$$

where l = original dimension,
α = coefficient of linear expansion,
and $\Delta\theta$ = *change* in temperature.

If a solid material is subjected to an increase in temperature $\Delta\theta$, and the resulting expansion is completely (or partially) restricted, a *compressive* stress will be induced in the material. Similarly, if there is a decrease in temperature and the resulting contraction is restricted, a *tensile* stress will be induced.

Figure 1.5(a) shows a rod with initial length l. If the temperature of the rod is increased from θ_1 to θ_2, the length of the rod will then be $(l + x)$, fig. 1.5(b), and the rod will remain unstressed. If the expansion is restricted as shown in fig. 1.5(c), then a compressive stress σ together with its associated strain will be induced in the rod, i.e. the *change* in temperature will *change* the stress from zero in fig. 1.5(a) to σ in fig. 1.5(c), provided that the expansion is restricted.

Referring to fig. 1.5(c), where the expansion is completely restricted,

$$\text{compressive strain } \epsilon = \frac{x}{l + x} = \frac{\sigma}{E}$$

$$\therefore \quad \sigma = \frac{Ex}{l + x}$$

But $x = l\alpha(\theta_2 - \theta_1) = l\alpha\Delta\theta$

$$\therefore \quad \sigma = \frac{El\alpha\Delta\theta}{l + l\alpha\Delta\theta} = \frac{E\alpha\Delta\theta}{1 + \alpha\Delta\theta}$$

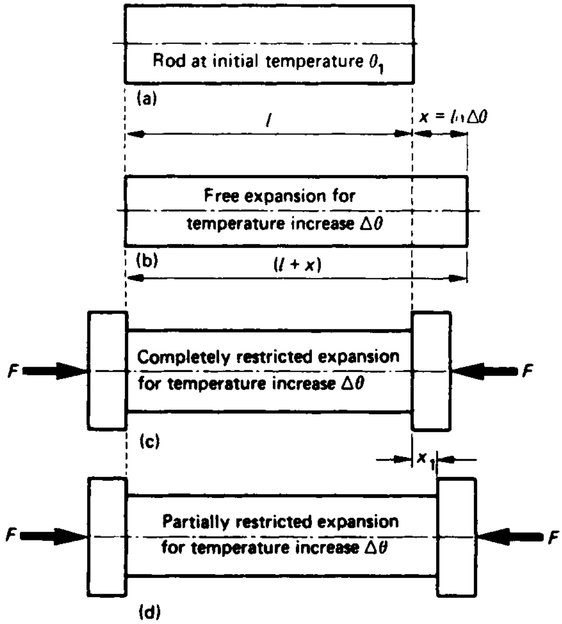

Fig. 1.5 Temperature stress

Since α is very small (typically 0.000 01), the term $\alpha\Delta\theta$ will be negligible compared with 1. Thus the term $(1 + \alpha\Delta\theta) \approx 1$; i.e., if the expansion is completely restricted, then a change in temperature $\Delta\theta$ will produce a change in stress of magnitude

$$\sigma = E\alpha\Delta\theta$$

which should be remembered.

Changes in stress which are induced by changes in temperature are known as temperature stresses.

Notice that $\alpha\Delta\theta = \sigma/E$

i.e. $\alpha\Delta\theta$ = *temperature strain*

which it is useful to remember.

If the expansion is restricted to an amount x_1 as shown in fig. 1.5(d), then

$$\text{total direct strain} = \frac{x - x_1}{l + x}$$

$$= \frac{l\alpha\Delta\theta - x_1}{l + l\alpha\Delta\theta}$$

$$= \frac{l\,[\alpha\Delta\theta - (x_1/l)]}{l\,(1 + \alpha\Delta\theta)}$$

As before, $(1 + \alpha\Delta\theta) \approx 1$

$\therefore$ total direct strain $= \alpha\Delta\theta - \dfrac{x_1}{l}$

= temperature strain – strain due to stress

But strain $= \sigma/E$

so, if the expansion is partially restricted, a change in temperature $\Delta\theta$ will produce a change in stress of magnitude

$$\sigma = E\left(\alpha\Delta\theta - \frac{x_1}{l}\right)$$

which should be remembered.

If a material is subjected to both an increase in temperature and a tensile stress (or a decrease in temperature and a compressive stress), then

total direct strain = temperature strain + strain due to stress

or total direct strain = sum of strains due to temperature change and stress

i.e. $$\epsilon = \alpha\Delta\theta \pm \frac{\sigma}{E}$$

which should be remembered.

Example 1 The temperature of a steel bush which is initially unstressed is increased from 20°C to 80°C. If the expansion is completely restricted, determine the compressive stress in the bush material. Take $E = 200\,\text{GN/m}^2$ and $\alpha = 12 \times 10^{-6}/°\text{C}$.

Since the expansion is completely restricted, the increase in temperature will induce a change in stress of magnitude

$$\sigma = E\alpha\Delta\theta$$

where $E = 200\,\text{GN/m}^2 = 200 \times 10^9\,\text{N/m}^2$ $\alpha = 12 \times 10^{-6}/°\text{C}$

and $\Delta\theta = 80°\text{C} - 20°\text{C} = 60°\text{C}$

$\therefore$ $\sigma = 200 \times 10^9\,\text{N/m}^2 \times 12 \times 10^{-6}/°\text{C} \times 60°\text{C}$

$= 144 \times 10^6\,\text{N/m}^2$ or $144\,\text{MN/m}^2$

i.e. the compressive stress in the bush material at 80°C is $144\,\text{MN/m}^2$.

Example 2 A spacer in a machine assembly was measured and found to be 200.00 mm long at a temperature of 20°C. After a period of time, the temperature of the spacer was found to be 60°C and its length 200.04 mm. Determine the compressive stress in the spacer at 60°C, if $E = 200\,\text{GN/m}^2$, $\alpha = 11.5 \times 10^{-6}/°\text{C}$, and the spacer was initially unstressed.

For partially restricted expansion, the increase in temperature will induce a change in stress of magnitude

$$\sigma = E\,(\alpha\Delta\theta - x_1/l)$$

where $E = 200\,\text{GN/m}^2 = 200 \times 10^9\,\text{N/m}^2$ $\alpha = 11.5 \times 10^{-6}/^\circ\text{C}$

$x_1 = 0.04\,\text{mm}$ $l = 200\,\text{mm}$ and $\Delta\theta = 60^\circ\text{C} - 20^\circ\text{C} = 40^\circ\text{C}$

$$\therefore \quad \sigma = 200 \times 10^9\,\text{N/m}^2 \left[(11.5 \times 10^{-6}/^\circ\text{C} \times 40^\circ\text{C}) - \frac{0.04\,\text{mm}}{200\,\text{mm}}\right]$$

$$= 200 \times 10^9\,\text{N/m}^2 \times 2.6 \times 10^{-4}$$

$$= 52 \times 10^6\,\text{N/m}^2 \quad \text{or} \quad 52\,\text{MN/m}^2$$

i.e. the compressive stress in the spacer at 60°C is $52\,\text{MN/m}^2$.

Example 3 The gauge length of an unstressed tensile-test specimen at a temperature of 20°C was 50 mm. If the tensile test is conducted when the temperature of the specimen is 80°C, estimate the gauge length for a tensile stress of $150\,\text{MN/m}^2$. For the material, $E = 200\,\text{GN/m}^2$ and $\alpha = 12 \times 10^{-6}/^\circ\text{C}$.

Total strain = temperature strain + strain due to stress

i.e. $\epsilon = \alpha\Delta\theta + \sigma/E$

where $\alpha = 12 \times 10^{-6}/^\circ\text{C}$ $\Delta\theta = 80^\circ\text{C} - 20^\circ\text{C} = 60^\circ\text{C}$

$\sigma = 150\,\text{MN/m}^2 = 150 \times 10^6\,\text{N/m}^2$ and $E = 200 \times 10^9\,\text{N/m}^2$

$$\therefore \quad \epsilon = 12 \times 10^{-6}/^\circ\text{C} \times 60^\circ\text{C} + \frac{150 \times 10^6\,\text{N/m}^2}{200 \times 10^9\,\text{N/m}^2}$$

$$= (0.72 \times 10^{-3}) + (0.75 \times 10^{-3})$$

$$= 1.47 \times 10^{-3}$$

Increase in length, $x = \epsilon l$

$$= 1.47 \times 10^{-3} \times 50\,\text{mm}$$

$$= 0.074\,\text{mm}$$

$\therefore$ final gauge length $= 50\,\text{mm} + 0.074\,\text{mm}$

$= 50.074\,\text{mm}$

i.e. the final gauge length is 50.074 mm.

1.7 Effect of temperature change on composite bars

Consider a composite bar consisting of a copper core A placed inside a steel tube B, with the end faces of the core and the tube being firmly attached to end plates as shown in fig. 1.6(a). When the assembly is at a temperature θ_1, A and B are of equal length l and are assumed to be unstressed.

If one of the end plates is removed and the temperature of the assembly is increased to θ_2, A and B will expand freely to the positions shown in

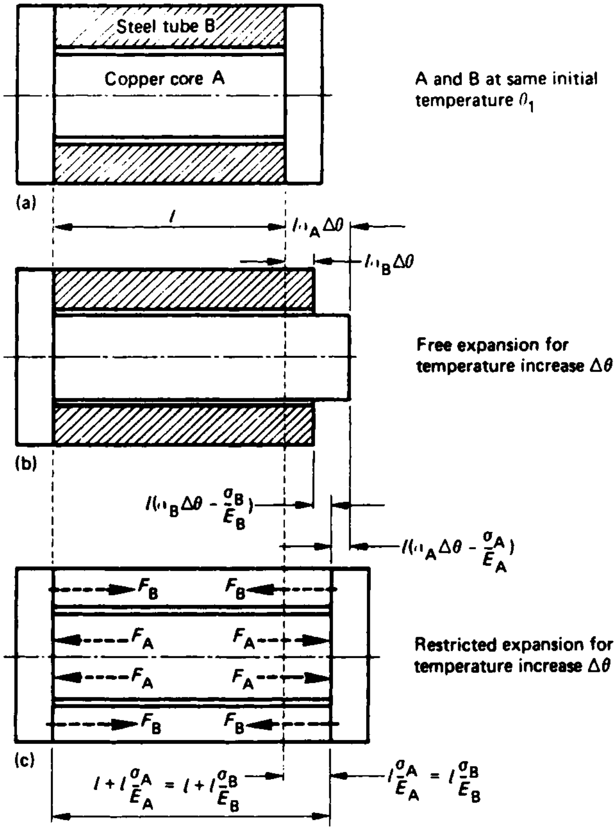

Fig. 1.6 Effect of temperature increase on a compound bar

fig. 1.6(b). Referring to fig. 1.6(b), the copper core A has expanded more than the steel tube B because the coefficient of linear expansion of copper is greater than that for steel.

If, with both end plates in position, the temperature of the assembly is again increased from θ_1 to θ_2, then, provided there is no distortion in the end plates, the copper and the steel will expand an *equal* amount as shown in fig. 1.6(c). Referring to fig. 1.6(c), the length of the copper core A is *reduced* and the length of the steel B is *increased* from the position shown in fig. 1.6(b); i.e. the copper core A is now being subjected to a *compressive* stress σ_A and the steel tube B to a *tensile* stress σ_B.

From section 1.6,

total direct strain = sum of strains due to temperature change and stress

or $$\epsilon = \alpha\Delta\theta \pm \sigma/E$$

For the copper core A,

$$\epsilon_A = \alpha_A \Delta\theta - \sigma_A / E_A$$

For the steel tube B,

$$\epsilon_B = \alpha_B \Delta\theta + \sigma_B / E_B$$

where $\Delta\theta = \theta_2 - \theta_1$

At temperature θ_2,

length of A = length of B

i.e. total direct strain in A, ϵ_A = total direct strain in B, ϵ_B

or $\alpha_A \Delta\theta - \sigma_A / E_A = \alpha_B \Delta\theta + \sigma_B / E_B$

$$\therefore \quad (\alpha_A - \alpha_B)\,\Delta\theta = \frac{\sigma_A}{E_A} + \frac{\sigma_B}{E_B}$$

i.e. difference between the temperature strains = sum of the strains due to the stresses

which it is useful to remember.

Referring to fig. 1.6(c), the copper core A is exerting a *pushing* force F_A on the end plates, while the steel tube B is exerting a *pulling* force F_B. Since the assembly is in equilibrium,

force exerted by A = force exerted by B

or $F_A = F_B$

If A_A and A_B represent the cross-sectional areas of A and B respectively,

then $F_A = \sigma_A A_A$ and $F_B = \sigma_B A_B$

$\therefore \quad \sigma_A A_A = \sigma_B A_B$

which it is useful to remember.

Example 1 If the diameter of the copper core in the composite bar shown in fig. 1.6(a) is 45 mm, and the internal and external diameters of the steel tube are 50 mm and 80 mm respectively, determine the stresses in the copper and the steel for a temperature increase of 60°C if both are assumed to be initially unstressed. For steel, $E = 200 \times 10^9\ \text{N/m}^2$ and $\alpha = 11.5 \times 10^{-6}/°\text{C}$. For copper, $E = 120 \times 10^9\ \text{N/m}^2$ and $\alpha = 16.5 \times 10^{-6}/°\text{C}$.

Let subscript A refer to the copper and subscript B refer to the steel.

$$\sigma_A A_A = \sigma_B A_B$$

$$\therefore \quad \sigma_A = \sigma_B (A_B / A_A)$$

where $A_A = (\pi/4) \times (45\ \text{mm})^2 = 1590\ \text{mm}^2$

and $A_B = (\pi/4) \times (80^2 - 50^2)\ \text{mm}^2 = 3063\ \text{mm}^2$

$$\therefore \quad \sigma_A = \frac{3063\,\text{mm}^2}{1590\,\text{mm}^2} \times \sigma_B = 1.93\,\sigma_B$$

$$(\alpha_A - \alpha_B)\,\Delta\theta = \sigma_A/E_A + \sigma_B/E_B$$

$$= 1.93\,\sigma_B/E_A + \sigma_B/E_B$$

i.e. $$\sigma_B = \frac{(\alpha_A - \alpha_B)\,\Delta\theta}{1/E_B + 1.93/E_A}$$

where $\alpha_A = 16.5 \times 10^{-6}/°\text{C}$ $\quad \alpha_B = 11.5 \times 10^{-6}/°\text{C}$ $\quad \Delta\theta = 60°\text{C}$

$E_A = 120 \times 10^9\,\text{N/m}^2$ and $E_B = 200 \times 10^9\,\text{N/m}^2$

$$\therefore \quad \sigma_B = \frac{(16.5 - 11.5) \times 10^{-6}/°\text{C} \times 60°\text{C}}{1.93/(120 \times 10^9\,\text{N/m}^2) + 1/(200 \times 10^9\,\text{N/m}^2)}$$

$$= \frac{3 \times 10^{-4}}{(1.608 \times 10^{-11}) + (0.5 \times 10^{-11})}\,\text{N/m}^2$$

$$= 14.23 \times 10^6\,\text{N/m}^2 \quad \text{or} \quad 14.23\,\text{MN/m}^2$$

and $$\sigma_A = 1.93\,\sigma_B$$

$$= 1.93 \times 14.23\,\text{MN/m}^2$$

$$= 27.46\,\text{MN/m}^2$$

i.e. the stresses in the steel and copper after the temperature increase are $14.23\,\text{MN/m}^2$ and $27.46\,\text{MN/m}^2$ respectively.

Example 2 The assembly shown in fig. 1.7 consists of a brass cylinder clamped between flanges by a steel stud. The cylinder is 60mm diameter x 45mm bore and the stud is 12mm diameter. At a temperature of 10°C, the tensile stress in the stud is $60\,\text{N/mm}^2$. Calculate (a) the compressive stress in the cylinder material at 10°C, (b) the stresses in the cylinder and stud materials when the temperature of the assembly is 40°C, (c) the stresses in the cylinder and stud materials when the assembly at a temperature of 40°C is subjected to an axial compressive force of 5kN. Ignore the effect of temperature on the flanges. For steel, $E = 200\,\text{kN/mm}^2$ and $\alpha = 11.5 \times 10^{-6}/°\text{C}$. For brass, $E = 90\,\text{kN/mm}^2$ and $\alpha = 17 \times 10^{-6}/°\text{C}$.

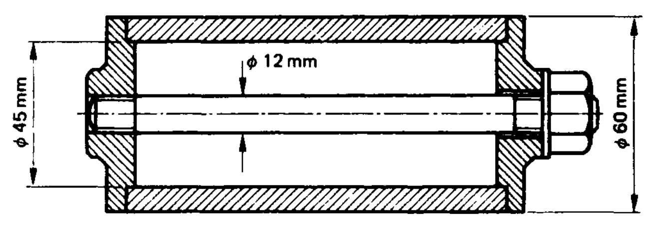

Fig. 1.7

Let subscript A refer to the brass and subscript B refer to the steel.
a) Referring to fig. 1.7,

force on cylinder = force on stud

i.e. $\sigma_A A_A = \sigma_B A_B$

or $\sigma_A = \sigma_B (A_B / A_A)$

where $A_A = (\pi/4) \times (60^2 - 45^2)\,\text{mm}^2 = 1237\,\text{mm}^2$

and $A_B = (\pi/4) \times (12\,\text{mm})^2 = 113.1\,\text{mm}^2$

$$\therefore \quad \sigma_A = \frac{113.1\,\text{mm}^2}{1237\,\text{mm}^2} \times \sigma_B$$

$$= 0.091\,\sigma_B$$

At 10°C,

$$\sigma_{B,10} = 60\,\text{N/mm}^2$$

$$\therefore \quad \sigma_{A,10} = 0.091 \times 60\,\text{N/mm}^2$$

$$= 5.46\,\text{N/mm}^2$$

i.e. at a temperature of 10°C, the compressive stress in the cylinder is 5.46 N/mm².

b) $(\alpha_A - \alpha_B)\,\Delta\theta = \sigma_A/E_A + \sigma_B/E_B$

$= 0.091\,\sigma_B/E_A + \sigma_B/E_B$

i.e. $$\sigma_B = \frac{(\alpha_A - \alpha_B)\,\Delta\theta}{0.091/E_A + 1/E_B}$$

where $\alpha_A = 17 \times 10^{-6}/°\text{C}$ $\qquad \alpha_B = 11.5 \times 10^{-6}/°\text{C}$

$\Delta\theta = 40°\text{C} - 10°\text{C} = 30°\text{C}$

$E_A = 90\,\text{kN/mm}^2 = 90 \times 10^3\,\text{N/mm}^2$

and $E_B = 200\,\text{kN/mm}^2 = 200 \times 10^3\,\text{N/mm}^2$

At 40°C,

$$\sigma_{B,40} = \frac{(17 - 11.5) \times 10^{-6}/°\text{C} \times 30°\text{C}}{0.091/(90 \times 10^3\,\text{N/mm}^2) + 1/(200 \times 10^3\,\text{N/mm}^2)}$$

$$= \frac{1.65 \times 10^{-4}}{(1.011 + 5) \times 10^{-6}}\,\text{N/mm}^2$$

$$= 27.45\,\text{N/mm}^2$$

i.e. the tensile stress in the stud is increased by 27.45 N/mm²,

$\therefore$ total stress in the stud at 40°C $= 60\,\text{N/mm}^2 + 27.45\,\text{N/mm}^2$

$= 87.45\,\text{N/mm}^2$

and total stress in the brass at 40°C = $87.45\,N/mm^2 \times 0.091$

$= 7.96\,N/mm^2$

i.e. the total stresses in the cylinder and stud materials at a temperature of 40°C are $7.96\,N/mm^2$ and $87.5\,N/mm^2$ respectively.

c) When the external force is applied,

strain in cylinder = strain in stud

or $\sigma_A/E_A = \sigma_B/E_B$

$\therefore \quad \sigma_A = \sigma_B(E_A/E_B)$

where $E_A = 90kN/mm^2$ and $E_B = 200kN/mm^2$

$$\therefore \quad \sigma_A = \frac{90kN/mm^2}{200kN/mm^2} \times \sigma_B$$

$$= 0.45\,\sigma_B$$

Also, total force = force on cylinder + force on stud

or $F = \sigma_A A_A + \sigma_B A_B$

$= 0.45\,\sigma_B A_A + \sigma_B A_B$

$$\therefore \quad \sigma_B = \frac{F}{0.45A_A + A_B}$$

where $F = 5 \times 10^3\,N$ $\quad A_A = 1237mm^2$ and $A_B = 113.1mm^2$

$$\therefore \quad \sigma_B = \frac{5 \times 10^3\,N}{0.45 \times 1237mm^2 + 113.1mm^2}$$

$$= 7.47N/mm^2$$

and $\sigma_B = 0.45 \times 7.47N/mm^2$

$= 3.36N/mm^2$

The effect of these compressive stresses due to the external force is to

i) *reduce* the tensile stress in the stud,
ii) *increase* the compressive stress in the cylinder.

$\therefore$ resultant stress in the stud = $87.45N/mm^2 - 3.36N/mm^2$

$= 84.09N/mm^2$ (tensile)

and resultant stress in the cylinder = $7.96N/mm^2 + 7.47N/mm^2$

$= 15.43N/mm^2$ (compressive)

i.e. the resultant stresses in the cylinder and stud materials are $15.43N/mm^2$ (compressive) and $84.09N/mm^2$ (tensile) respectively.

Referring to the above example, notice that stresses induced by external forces and by changes in temperature can be combined, provided that due care is taken with the magnitude and sense (or nature) of the respective stresses.

1.8 Shear stress

Shear stress is defined as shear force F per unit cross-sectional area A resisting the shear,

i.e. $$\text{shear stress} = \frac{\text{shear force}}{\text{cross-sectional area resisting shear}}$$

The symbol used for shear stress is τ (*tau*),

i.e. $$\tau = \frac{F}{A}$$

which should be remembered.

The basic unit for shear stress is the newton per square metre (N/m^2).

Example 1 The simple riveted lap joint shown in fig. 1.8 contains six rivets, each 8 mm diameter, and is subjected to a shear force of 1.6 kN. Determine the shear stress in each rivet.

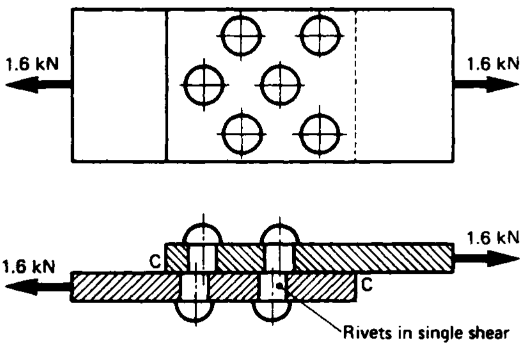

Fig. 1.8 Simple riveted lap joint

Referring to fig. 1.8, the joint will shear at the interface CC,

i.e. area resisting shear, A = total cross-sectional area of all the rivets

$= (\pi/4) \times (8\,\text{mm})^2 \times 6$ rivets

$= 301.6\,\text{mm}^2$

Shear stress $\tau = F/A$

where $F = 1.6\,\text{kN} = 1600\,\text{N}$